Golomb's Puzzle Column™

An Ingenuity Test

Four of these problems are modifications and generalizations of ones I found in the December, 2000, issue of the Canadian journal *Crux Mathematicorum*, which mainly features problems from high school mathematics competitions. However, problem 3 was recently told to me by Robert M. Stewart, who had just discovered it. (Is it new? Is it true? Can you prove it?) and problem 4 was first solved by John L. Selfridge in 1955. Send me any clever solutions you find, particularly of problems 2, 3, and 4.

1. Simultaneous Equations
Solve for $x$ and $y$ in terms of $a$ and $b$. (Assume that $a$ and $b$ are positive real numbers.)

\[
\begin{align*}
{x^2 + xy + x} &= a \\
{y^2 + xy + y} &= b
\end{align*}
\]

2. How Many Triangles?
A total of $n(n+1)/2$ points are arranged in an equilateral triangle, $n$ on a side, in the usual way. For $n=4$, the arrangement looks like:

![Diagram of an arrangement of points in an equilateral triangle]

We ask for the number $f(n)$ of different subsets consisting of three of these points which are at the vertices of an equilateral triangle. It is easily verified that $f(1)=0$, $f(2)=1$, $f(3)=5$, and $f(4)=15$. Prove or disprove: $f(n) = \left(\frac{n+2}{4}\right)$ for all $n \geq 1$.

3. Thinking Inside the Box
We are given an $m \times n$ “square array” of dots, and are required to draw a continuous path $P$ from the upper left corner to the lower right corner, consisting entirely of straight line segments, which goes through all the dots, stays inside the $m \times n$ rectangle, and never intersects itself. We can then connect the two endpoints of the path by a simple curve completely outside the “box” (the $m \times n$ rectangle), and by the Jordan Curve Theorem the resulting simple closed curve partitions the plane into an “inside” and an “outside”. Color the inside blue and the outside red. Show that, inside the “box”, the blue area equals the red area, no matter how the path $P$ was drawn.

Example:

![Diagram of a path inside a box with blue and red areas]

Red area (R) = Blue area (B) = 6 units.

4. Thinking Outside the Box
On an $m \times n$ “square array” of dots (as in problem 3) we wish to draw a continuous path $P$, consisting entirely of straight line segments, which goes through every dot at least once. The path $P$ is allowed to go outside the $m \times n$ rectangle, and may intersect itself. What is the smallest number of straight line segments that $P$ must contain, in terms of $m$ and $n$?

*Note:* A famous special case is $m = n = 3$, with a path $P$ consisting of only four segments:

![Diagram of a path outside a box]

Remember that mathematical points and lines have no thickness.

5. A Cubic Relation
Let $g(x)$ be a cubic polynomial with non-zero roots $r_1, r_2, r_3$, such that $g(\alpha) + g(-\alpha) = K$ with real $\alpha \neq 0$ and real $K \neq 0$.

Find the value of $\frac{1}{r_1 r_2} + \frac{1}{r_2 r_3} + \frac{1}{r_3 r_1}$ in terms of $\alpha$ and $K$.

6. A Continued Fraction
Suppose that $A$, $B$, and $C$ are positive integers with $A + \frac{1}{B + \frac{1}{C \pi}} = \frac{115}{36}$. Find $A^2 + B^2 + C^2$. 

March 2001
The 6th Winterschool on Coding and Information Theory took place December 17-20 in the beautiful setting of Castle Reisensburg in Günzburg, Germany. The 50 participants enjoyed the exclusive atmosphere in the old completely renovated castle. The main goal of the Winterschool is that Ph.D. students from different universities get to know each other and that scientific presentations have an informal character. The tradition of the Winterschool is that only Ph.D. students (or just finished) are invited to give a 20 minute presentation. About 25 contributions were delivered from the Universities of Ulm, München, Erlangen, Essen, Kiel, Bremen in Germany, Linköping and Lund in Sweden, Bergen in Norway and Zürich and Lausanne in Switzerland. Four main lectures of a tutorial nature were given by Johannes Huber (Erlangen) on Multilevel Codes, Amos Lapidoth (Zürich) on Limits on Reliable Communication over Flat-Fading Channels, Jim Massey (Lund) on Noisy Sequence Acquisition and Viktor Zyablov (Moscow) on the History of Concatenated Codes. Earlier workshops took place in Essen (Germany, 1991 and 1993), Eindhoven (the Netherlands, 1994), Mölle (Sweden, 1996) and Aarhus (Denmark, 1998). In addition Prof. Fliedner gave a 2 hour tour on the history and development of the castle. The meeting was sponsored by the IEEE German chapter on Information Theory and the University of Ulm. Information on the meeting can be obtained from the organizer Martin Bossert (University of Ulm).

The 23rd Symposium on Information Theory and its Applications SITA 2000

Aso, Kumamoto, Japan
October 10-13,

The year 2000 symposium was organized by Kyoki Imamura, from Kyushu Institute of Technology, in the beautiful setting of Kumamoto. The Greenpia hotel is one of the most spectacular places for a conference I have ever seen. The location is famous for its vulcan “Aso” area. In this respect it is a perfect match to the Sorrento IEEE-ISIT symposium. About 300 participants and 250 presented papers make this yearly symposium the biggest non-IEEE Information Theory event. A great problem for non-Japanese participants is the language. Although there are some (2) English sessions, the organization is completely focussed on the local Japanese community. In my view, the symposium can be of great importance for the Asian area if more attention is paid to the foreigners. Furthermore, many young people partici-
First Hong Kong Information Theory Colloquium (ITC) -

Hong Kong
October 28, 2000

Organized by Wai Ho Mow and Ted Kok, the first Hong Kong Information Theory Colloquium was held at the Hong Kong University of Science and Technology (HKUST) on October 28, 2000. After a warm welcome by Prof. Kang Wang and Prof. Philip Chan from HKUST, the opening addresses for the colloquium were delivered by Mr. Y. W. Liu, Deputy Chair of the IEEE Hong Kong Section, and Vijay Bhargava, President of the Society. Technical talks were delivered by Victor O. K. Lee, Mordecai J. Golin, Ping Wah Wong, Raymond Yeung, Lei Wei, Ping Li, and Roger Cheng. The colloquium was well attended by local researchers and students in information theory and related areas.

The IEEE Information Theory Hong Kong Chapter, organized by Raymond Yeung, was formed at the same event which was co-sponsored by the Chapter. The other founding committee members of the Chapter are Wai Ho Mow (vice-chairman), Tat-ming Lok (Honorary Secretary), Ted Kok (Honorary Treasurer), Mordecai Golin, Muzhong Wang, and Victor Wei. The Chapter has about 50 members.
CALL FOR PAPERS

2002 IEEE International Symposium on Information Theory

Palais de Beaulieu, Lausanne, Switzerland
June 30 – July 5, 2002

The 2002 IEEE International Symposium on Information Theory will be held at the Palais de Beaulieu in Lausanne, Switzerland, from Sunday, June 30, through Friday, July 5, 2002.

Previously unpublished contributions to the following areas are solicited:

- Coded modulation
- Coding theory and practice
- Communication complexity
- Communication systems
- Cryptology
- Data compression
- Data networks
- Detection and estimation
- Information theory and statistics
- Multiuser detection
- Multiuser information theory
- Pattern recognition and learning
- Quantum information processing
- Shannon theory
- Signal processing
- Source coding

Papers will be reviewed on the basis of an extended summary (not exceeding six pages) of sufficient detail to permit reasonable evaluation. The deadlines for submission is September 30, 2001 for paper copies and October 7, 2001 for electronic copies, with notification of decisions by February 8, 2002. In view of the large number of submissions expected, multiple submissions by the same author will receive especially stringent scrutiny. All accepted papers will be allowed twenty minutes for presentation, and one-page abstracts will be printed in the conference proceedings. Authors are strongly encouraged to submit electronic versions of their summaries by following the guidelines on the symposium web page. For those unable to submit electronically, four copies of the summary should be mailed to

Ms. Monique Borcard
ISIT 2002 Paper Submission
EPFL — DSC — LTHI
CH-1015 Lausanne
Switzerland

It is expected that a small number of grants for the partial reimbursement of travel costs may be available for the authors of accepted papers whose resources would not otherwise enable them to attend the symposium. Detailed information on the technical program, special events, accommodations, travel arrangements, excursions and applications for travel grants will be included in subsequent mailings, and will be posted at the symposium web site

http://isit02.epfl.ch

Inquiries on general matters related to the symposium should be addressed to

Prof. Bixio Rimoldi
Communication Systems Department
Swiss Federal Institute of Technology
CH-1015 Lausanne, Switzerland
E-mail: isit02chair@epfl.ch
Phone: +41 21 693 76 62
Fax: +41 21 693 43 12
The Thirty-Ninth Annual Allerton Conference on Communication, Control, and Computing will be held from Wednesday, October 3 through Friday, October 5, 2001, at the Allerton House, the conference center of the University of Illinois. Allerton House is located twenty-six miles southwest of the Urbana-Champaign campus of the University, in a wooded area on the Sangamon River. It is part of the fifteen-hundred acre Robert Allerton Park, a complex of natural and man-made beauty designated as a National natural landmark. The Allerton Park has twenty miles of well-maintained trails and a living gallery of formal gardens, studded with sculptures collected from around the world.

Papers presenting original research are solicited in the areas of communication systems, communication and computer networks, detection and estimation, information theory and error-correcting codes, source coding and data compression, multiple-access communications, queueing networks, control systems, robust and nonlinear control, adaptive control, optimization, dynamic games, large scale systems, robotics and automation, manufacturing systems, discrete event systems, intelligent control, multivariable control, computer vision based control, learning theory, neural networks, VLSI architectures for communications and signal processing, and automated highway systems. Also solicited are organized sessions for the Conference; prospective organizers should discuss their plans with the Conference co-chairs before sending a formal proposal.

This year the plenary lecture will be delivered by Professor John C. Doyle of the California Institute of Technology. It is scheduled for Friday, October 5, and is entitled “Robustness and Network Complexity.”

Information for authors: Regular papers, suitable for presentation in twenty minutes, as well as short papers, suitable for presentation in ten minutes, are solicited. The purpose of the short paper category is to encourage authors to present preliminary results of their work. Regular papers will be published in full (subject to a maximum length of ten 8.5” x 11” pages) in the Conference Proceedings, while short papers will be limited to two-page summaries in the Proceedings.

For regular papers, a title and a five-to-ten page extended abstract, including references and sufficient detail to permit careful reviewing, are required. For short papers, a title and a three-to-five page summary are required. Manuscripts that are submitted as regular papers but cannot be accommodated in that category will be considered in the short paper category, unless the authors indicate otherwise.

Three copies of the manuscript should be mailed to 39th Annual Allerton Conference, Coordinated Science Laboratory, University of Illinois, 1308 West Main Street, Urbana, Illinois 61801-2307, USA, in time to be received by July 6, 2001. Submissions by e-mail or fax will not be accepted.

Submissions should specify the name, e-mail address, and postal address of the author who is to receive all subsequent correspondence. Authors will be notified of acceptance via e-mail by August 10, 2001, at which time they will also be sent detailed instructions for the preparation of their papers for the Proceedings. Full camera-ready versions of accepted papers will be due the last day of the Conference.

Conference Co-Chairs: Douglas L. Jones and Petros G. Voulgaris
Email: allerton@csl.uiuc.edu URL: http://www.comm.csl.uiuc.edu/allerton

COORDINATED SCIENCE LABORATORY AND
THE DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
University of Illinois at Urbana-Champaign
GOLOMB’S PUZZLE COLUMN™ SOME PARTITION PROBLEMS SOLUTIONS

Let \( N_s(n,k) \) denote the number of representations of \( n \) as a sum of (exactly) \( k \) positive integers, in each of four situations:

\[
\begin{align*}
    x &= a. \text{ The summands are distinct, and order doesn’t matter.} \\
    x &= b. \text{ The summands are distinct, and order matters.} \\
    x &= c. \text{ The summands needn’t be distinct, and order doesn’t matter.} \\
    x &= d. \text{ The summands needn’t be distinct, and order matters.}
\end{align*}
\]

The simplest general formula is \( N_d(n,k) = \binom{n-1}{k-1} \). To show this, imagine \( n-k \) consecutive dashes (“-”), and \( k \) composite symbols of the type “1-”. One composite symbol is placed at the left end of the sequence of dashes, and the other \( k-1 \) composite symbols are inserted arbitrarily into the sequence, allowing but not requiring two or more in a row. What results is a sequence with \( n \) horizontals (dashes) and \( k \) verticals (“1”-symbols). Starting at the left end (a vertical), we count the number of dashes between consecutive verticals, imagining a final vertical at the end. This gives a sequence of positive integers summing to \( n \), obtained in \( \binom{n-k+1}{k-1} \) different ways.

It is easy to get the exact relationships among \( N_a(n,k), N_b(n,k), \) and \( N_c(n,k) \). Obviously, \( N_b(n,k) = k! N_a(n,k) \), since all \( k \) summands are distinct in these cases, and these are accordingly \( k! \) permutations of them.

Somewhat more surprising is the relation \( N_a(n,k) = N_c(n-k\frac{2}{2},k) \). To see this, imagine \( n \) written as the sum \( a_1 + a_2 + \ldots + a_k \) with \( 1 \leq a_1 \leq a_2 \leq \ldots \leq a_k \), which counts toward \( N_c(n,k) \). Then \( n + \binom{k}{2} = (a_1 + 1) + (a_2 + 1) + (a_3 + 2) + \ldots + (a_k + k - 1) \), which counts toward \( N_c(n + \binom{k}{2},k) \), and this correspondence is one-to-one. Thus, if we kno...


*Contents:*
Introduction; Basic Concepts and Approaches; Single Input Single Output Blind Equalization Algorithms; Local Convergence Analysis of SISO Blind Equalizers; Linear Multichannel Blind Identification from Second Order Statistics; Frequency Domain Approach to Single User Channel Identification; Adaptive Multichannel Equalization; Selected Topics in Multichannel Equalization.


*Contents:*
Voice Communications: Advanced Intelligent Networks; Computer Telephony Integrated (CTI); Voice over IP; Internet: Internet and Intranet Management Concepts; Internet Security; Virtual Private Networks; Effective Web Site Design; Web Enables Data Warehouse; E-Commerce Technologies; Internet Protocols. Management and Administration: Management Concepts; Management of Emerged and Emerging Technologies; Commercial Network and Systems Management Standards; Telecommunications Network Management (TMN); Telecommunications Information Networking Architecture (TINA); Telecommunications Support Process; Management Frameworks and Applications; Customer Network Management; Aspects of Managing Outsourcing Solutions; Support Systems for Telecommunication Providers; Performance Management of Intranets. Future Telecommunications: Trends and Directions.


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**Conference Calendar**

**March 19-29, 2001**  
*Nonlinear Estimation and Classification*  
Mathematical Sciences Research Institute  
1000 Centennial Drive  
Berkeley, CA 94720-5070  
Email: workshops@msri.org  
Web: http://www.msri.org

**March 21-23, 2001**  
*35th Annual Conference on Information Sciences and Systems — CISS 2001*  
Johns Hopkins University  
105 Barton Hall  
105 Barton Hall  
Department of Electrical and Computer Engineering  
Johns Hopkins University  
Baltimore, MD 21218  
Tel: 410-516-7033  
Fax: 410-516-5566  
Web: http://www.ece.jhu.edu/ciss

**March 13-17, 2001**  
*Sequences and Their Applications — SETA’01*  
P. Vijay Kumar  
Communication Sciences Institute  
Electrical Engineering Systems  
University of Southern California  
Los Angeles, CA 90089-2565  
Tel: +1-213-740-4668  
Fax: +1-213-740-8729  
Email: vijayk@usc.edu  
Web: http://www.ii.uib.no/seta01

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March 2001  
IEEE Information Theory Society Newsletter
Conference Calendar

June 3-6, 2001  British Columbia, Vancouver

2001 Canadian Workshop on Information Theory
Due Date: January 8, 2001
Dr. C. S. Leung
Department of Electrical & Computer Engineering
University of British Columbia
2356 Main Mall
Vancouver, B.C., V6T 1Z4
Tel: +1-604-822-2866
Fax: +1-604-822-5949
Email: cleung@ece.ubc.edu
Web: http://datacom.ece.ubc.ca/cwit

June 10-16, 2001  Sunny Beach, Bulgaria

3rd International Workshop on Optimal Codes and Related Topics
Due Date: January 31, 2001
Dr. Ivan Landjev
Institute of Mathematics and Informatics
8 Acad G. Bonchev Str.
1113 Sofia, BULGARIA
Tel: +359-2-979-2821
Fax: +359-2-971-3649
Email: oc2001@moi.math.bas.bg

June 24-29, 2001  Washington, D.C., USA

IEEE International Symposium on Information Theory — ISIT 2001
Due Date: October 1, 2000
Prof. Prakash Narayan
Department of Electrical and Computer Engineering
University of Maryland
College Park, MD 20742 USA
Tel: (301) 405-3661
Fax: (301) 314-9281
Email: prakash@eng.umd.edu
Web: http://www.seas.smu.edu/isit2001/

September 2-7, 2001  Cairns, Australia

2001 IEEE Information Theory Workshop
Due Date: March 31, 2001
Dr Lei Wei
School of Elec., Comp. & Telecommun. Eng
University of Wollongong
NSW 2522, Australia
L.Wei@elec.uow.edu.au
Phone: +61 2 4221 3407
Fax: +61 2 4221 3236

October 3-5, 2001  Essen, Germany

Mini-Workshop on Convolutional Codes
Due Date: August 7, 2001
Han Vinck
IEM
Ellernstrasse 29
45326 Essen, Germany
Fax: +49 201 183 7663
E-mail: vinck@exp-math.uni-essen.de

June 30-July 5, 2002  Palais de Beaulieu, Lausanne, Switzerland

2002 IEEE International Symposium on Information Theory
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IEEE Information Theory Society Newsletter

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